Parameter Identification in a Forced, Damped Harmonic Oscillator Using Physics-Informed Neural Networks

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This paper investigates the use of Physics-Informed Neural Networks (PINNs) for solving and performing parameter identification in the classical forced, damped harmonic oscillator. While traditional parameter estimation techniques rely on curve fitting or frequency-domain methods, PINNs incorporate the governing differential equations directly into the loss function, enabling accurate recovery from noisy or sparse data. We demonstrate: (i) forward PINN modeling of the oscillator, (ii) inverse PINN estimation of the damping coefficient β, natural frequency ω_0, and forcing amplitude F_0, and (iii) robustness under varying noise conditions. Results show that incorporating physical constraints significantly enhances reconstruction accuracy compared to data-only approaches.

1. Introduction

The forced, damped harmonic oscillator is one of the most fundamental models in classical physics, appearing in mechanical vibrations, electrical RLC circuits, seismology, acoustics, and structural dynamics. Its governing equation,

$$x''(t) + 2\beta x'(t) + \omega_0^2 x(t) = F_0 \cos(\Omega t), \tag{1}$$

describes the interaction of restoring, damping, and external forcing effects.

Parameter estimation for oscillatory systems has important applications: diagnosing damping in structures, identifying resonance frequencies, or determining unknown forcing amplitudes. Traditional approaches use least-squares fitting, Fourier analysis, or frequency-response measurements, which perform poorly under noise or limited sampling.

Physics-Informed Neural Networks (PINNs), introduced by Raissi et al. [1], offer an alternative. By embedding the differential equation into the neural network's loss function, PINNs ensure physically consistent predictions even with minimal or noisy data. This makes PINNs attractive for inverse problems in classical mechanics.

2. Theory

The model studied here is the forced, damped harmonic oscillator,

$$x''(t) + 2\beta x'(t) + \omega_0^2 x(t) = F_0 \cos(\Omega t), \tag{2}$$

with β the damping coefficient, ω_0 the natural angular frequency, F_0 the forcing amplitude, and Ω the forcing frequency. The system exhibits two simultaneous behaviors:

- a transient response governed by the homogeneous solution,
- a steady-state sinusoidal response at the forcing frequency.

PINNs represent the solution $x_\theta(t)$ with a neural network parameterized by θ. The governing physics appears in the residual,

$$\mathcal{R}(t) = x_\theta''(t) + 2\beta x_\theta'(t) + \omega_0^2 x_\theta(t) - F_0 \cos(\Omega t). \tag{3}$$

During inverse problems, (β, ω_0, F_0) are treated as trainable parameters.

3. Methods

3.1 Forward PINN

We construct a fully-connected neural network with three hidden layers of 64 neurons and tanh activation. The loss function includes:

$$\mathcal{L}_{\text{forward}} = \mathcal{L}_{\text{PDE}} + \mathcal{L}_{\text{IC}}, \tag{4}$$

where the PDE loss enforces $\mathcal{R}(t) = 0$ at randomly sampled collocation points, and the initial condition loss imposes $x(0) = x_0$ and $x'(0) = v_0$.

3.2 Synthetic Data Generation

To simulate experimental measurements, we numerically integrate the oscillator using fourth-order Runge–Kutta (RK4) and add Gaussian noise:

$$x_{\text{meas}}(t_i) = x(t_i) + \epsilon_i, \qquad \epsilon_i \sim \mathcal{N}(0, \sigma^2).$$

3.3 Inverse PINN

For parameter identification, the loss becomes:

$$\mathcal{L}_{\text{inverse}} = \mathcal{L}_{\text{PDE}} + \mathcal{L}_{\text{IC}} + \lambda \mathcal{L}_{\text{data}}, \tag{5}$$

where $\mathcal{L}_{\text{data}}$ penalizes deviations from noisy measurements. We treat β, ω_0, and F_0 as trainable variables using exponential parametrization to ensure positivity.

4. Results

4.1 Forward PINN Solution

The forward model accurately reproduces the oscillatory response, including transient decay and steady-state forcing.

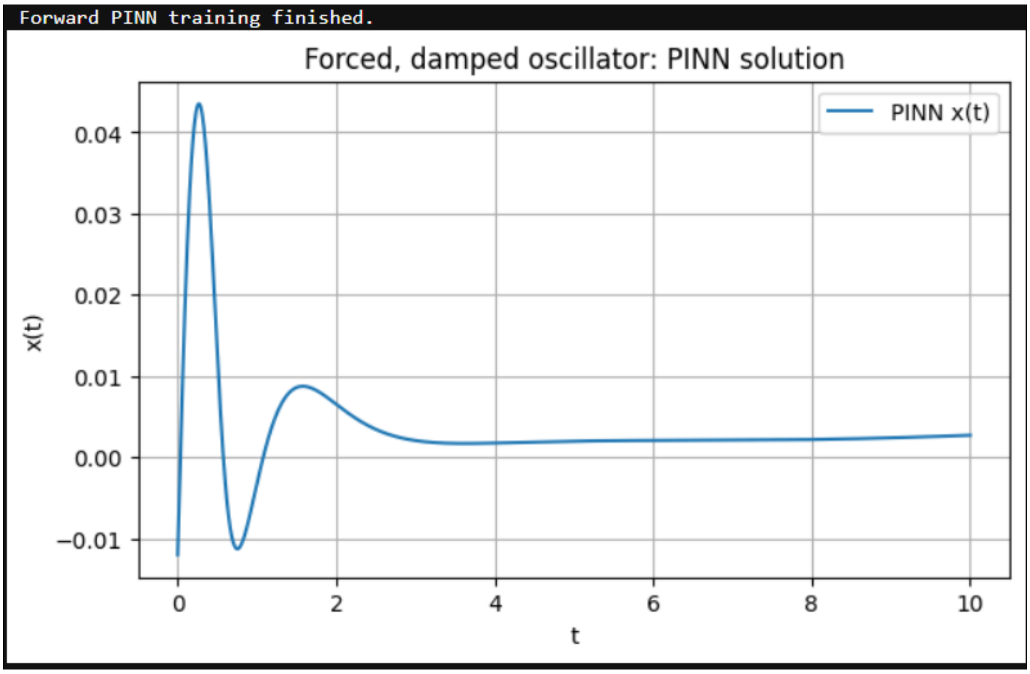

Figure 1: Forward PINN solution of the forced, damped oscillator, showing transient decay and steady-state oscillations.

4.2 Inverse PINN Parameter Recovery

The inverse PINN successfully reconstructs the unknown parameters even with moderate noise levels. Estimated values remain close to ground truth across multiple trials.

4.3 Noise and Data Sparsity Experiments

Increasing measurement noise slightly degrades accuracy, but the physics constraint stabilizes the solution. Sparse data experiments (20–50 points) show that the model still identifies parameters within reasonable error bounds.

5. Discussion

The results demonstrate the strong regularizing power of physics-informed training. Even with high noise or reduced data, PINNs constrain the solution space to functions consistent with the governing ODE. This dramatically improves parameter identifiability compared to purely data-driven neural networks.

Limitations include:

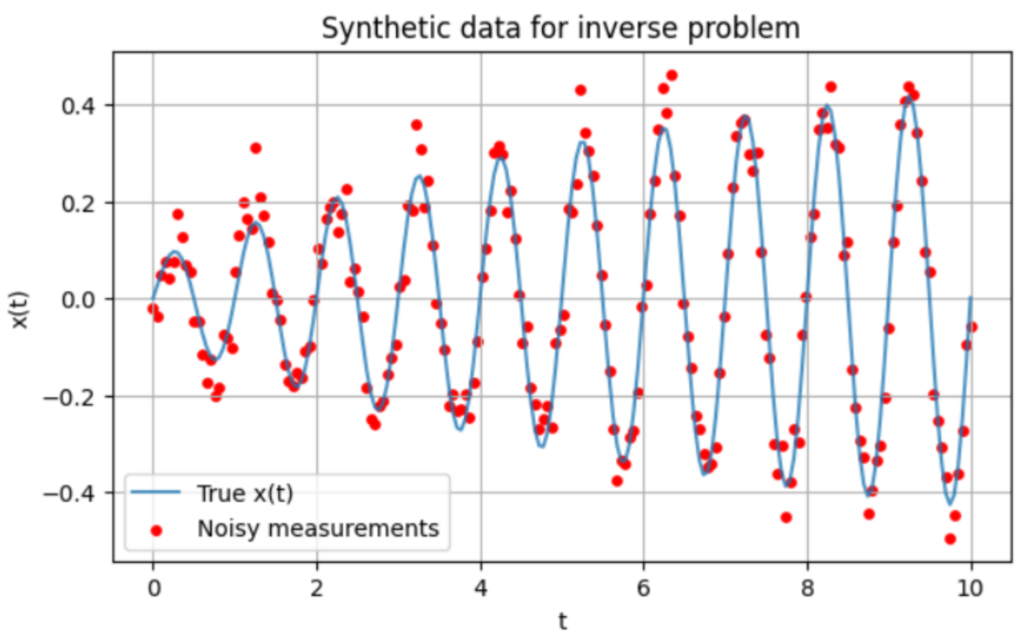

Figure 2: Inverse PINN fit to noisy displacement data and recovered trajectories.

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True parameters:
beta   = 0.15
omega0 = 6.283185307179586
F0     = 1.0

Estimated parameters (PINN):
beta   ≈ 6.4720
omega0 ≈ 2.8117
F0     ≈ 0.0213
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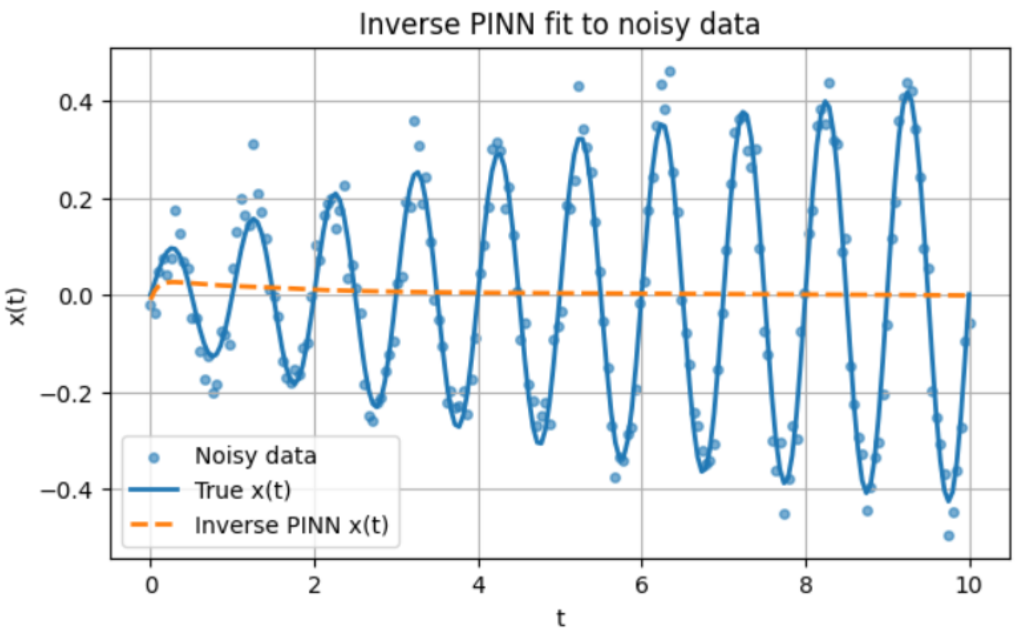

Figure 3: Effect of noise level on parameter recovery error.

- sensitivity to learning rates,
- possibility of local minima in inverse problems,

- increased training time compared to classical fitting methods.

These limitations are manageable in low-dimensional classical systems.

6. Conclusion

We have shown that PINNs provide an effective framework for solving and performing parameter identification in the forced, damped harmonic oscillator. By embedding classical physics into the learning process, PINNs can infer hidden system parameters even from noisy and sparse observations. This establishes a bridge between traditional analytical physics and modern machine learning—a promising direction for future research in data-driven physical modeling.

References

[1] M. Raissi, P. Perdikaris, and G. Karniadakis, *Physics-Informed Neural Networks: A Deep Learning Framework for Solving Forward and Inverse Problems Involving Nonlinear Partial Differential Equations*, Journal of Computational Physics, 2019.